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## Bidding Behavior in a Multi-attribute First-price Auction

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Reviewed, Approved, and Released by  
David M. Cashbaugh  
Director

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## Foreword

Using an experimental bidding environment, this research extends the concepts of a first-price sealed bid auction by incorporating a multi-attribute component where market participants set valuations on the attributes of a hypothetical job and explore the feasibility of applying key features of the multi-unit auction to proxy buyer/seller marginal valuations of the attributes of a job.

Two experiments were executed to observe bidding behavior in a multi-attribute auction setting over varying reserve prices and seller values. Interestingly, convergence of subject bids to individual reserve prices generally occurs within five auctions and even with as few subjects/bidders as seven.

While theoretically it can be shown that a first-price open out cry auction quickly converges to subject reserve prices, the first-price sealed bid multi-attribute auction design addressed in this paper also quickly converges to the subject reserve. Interestingly, the rapid convergence occurs even in cases where seller values differ across auctions. Applications of a multi-attribute auction to labor markets, where participants bid on multiple components of a compensation package show promise in ascertaining buyer/seller marginal valuations of a job.

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DAVID M. CASHBAUGH  
Director

# Contents

<b>Introduction: Multi Unit Auctions and Multi Dimensional Characteristics of Labor Market Services.....</b>	<b>1</b>
Multi-Attribute Labor Market Auction Model	2
Multi-Attribute Labor Market Auction Model: Scoring Rule	4
Experimental Design	6
Auction Results	8
<b>Conclusion.....</b>	<b>16</b>
<b>References .....</b>	<b>17</b>

## List of Tables

1. Rated seller .....	5
2. Experiment 1 parameters .....	7
3. Experiment 2 parameters .....	8

## List of Figures

1. Labor market auction interface. ....	3
2. Experiment 1 median of difference in composite bids. ....	9
3. Experiment 1 minimum of differences in the composite bid. ....	9
4. Experiment 2 median difference of composite bid. ....	10
5. Experiment 2 minimum of composite bid difference. ....	11
6. Experiment 2 Median difference between reserve price and bid for A1. ....	12
7. Experiment 2 minimum difference between reserve price and bid for A1. ....	13
8. Experiment 2 median difference between reserve price and bid for A2. ....	13
9. Experiment 2 minimum value of differences between reserve price and bid for A2. ....	14
10. Experiment 2 median of differences between reserve price and bid for A3. ....	14
11. Experiment 2 minimum of differences between reserve price and bid for A3. ....	15
12. Median of difference of composite bid by seller value. ....	16

## Introduction: Multi Unit Auctions and Multi Dimensional Characteristics of Labor Market Services

Standard auctions typically involve the auction of a single unit of a good or service; bids of monetary value are placed, and the good or service is awarded to the highest bidder, with auctions providing efficiency expected from standard economic theory (Bulow and Roberts, 1989). The efficiency of simple auctions was established by Vickrey (1961). In contrast, Multi-unit auctions have been successfully adopted for use in the goods market, (Matsui & Watanabe, 2003), with such auctions obtaining an efficient outcome when an equilibrium exists. The efficiency gains of multi-unit auctions observed in the goods market, as measured by maximizing consumer and producer surplus, are likely to have promising applications to labor markets. Multi-unit auctions allow for bundles of goods to be sold as a single unit, which can provide benefits for sellers who seek to practice price discrimination and also provides benefits to the volume purchaser who can obtain the savings of quantity discounts (Avery and Hendershott).

Sellers of standard goods and services may seek to bundle some products in such a way as to prevent the buyer from substituting inferior components. An example of such bundling is when an automobile is sold by a dealership with a 100,000 mile warranty, which bundles some service by the dealership with the automobile being sold. In the absence of the warranty, the customer may seek a low-cost and low-quality service mechanic, and blame poor performance of the car on the manufacturer, rather than on the low quality service of the low-cost mechanic. Similarly, employers may bundle compensation packages in such a way that employees may not be able to fully substitute among the compensation components. Health insurance may be valued differently among employees, such that younger and healthier employees would value the insurance less than the older and less healthy workers. However, when employers do offer flexible compensation packages, employees much choose among varying levels of the components, with the accompanying trade-offs involved.

In labor economics, the theory of compensating wage differentials emphasizes the relationship between the wage and non-pecuniary attributes of a job, where the simple model treats the wage as being determined by heterogenous employees who consider tradeoffs of wages and job related variables such as risk of injury, unpleasantness, etc. In that model, the movement of employees between different firms is similar to bidders choosing which bundles (each bundle having different quantities of the component attributes) to purchase. The more complete description of the co-determination of wages and job attributes is captured in hedonic models; where employees with heterogenous job attribute utility functions choose between firms which offer differing combinations of job attributes and pay. Heterogenous firms will have differing costs of producing attribute combinations, and the resulting isoprofit curves will present workers with different opportunity sets for each firm. In this way, firms and workers engage in a matching process which efficiently sorts both into the variety of jobs and attributes that we observe in the job market. In this type of analysis, there is only one good or service, but multiple attributes of that good or service, as contrasted with the multiple units of the previous auctions.

In the hedonic model, the ability of the system to achieve some degree of efficiency hinges upon having sufficient numbers of differentiated firms to offer a variety of wage/attribute combinations to the different workers. However, if firms have fairly consistent job descriptions *within* firms, then the number of firms becomes a limiting factor in obtaining an efficient match for the system, as the variety of wage/attribute combinations could only be accomplished by *between* firm differences in jobs.

If a sufficiently large firm operates in a labor market, it may be possible for that single firm to offer a variety of wage/attribute combinations if that firm can elicit information from workers about their preferences for various wage/attribute combinations. In doing so, the firm should be able to offer employees an opportunity to move to higher indifference curves while the firm offers jobs on its isoprofit curve. Instead of workers moving to higher indifference curves by moving between firms, the worker can potentially find the most desired wage/attribute combination without incurring the cost of moving to another firm. The firm which can offer such flexibility may enjoy an advantage in recruiting quality workers as well as lower turnover costs. This may require a variety in job descriptions and administration that may not be possible in a practical sense, even if the theoretical possibility exists. Large organizations tend to be characterized by human resource departments that adopt employment policies that involve ease of administration rather than organizational efficiency. However, we will explore the theoretical possibility for improving organizational efficiency, and leave the potential administrative barriers for other researchers.

Using an experimental bidding environment, this research extends the concepts of a first-price sealed bid auction by incorporating a multi-*attribute* component where market participants set valuations on the attributes of a hypothetical job and explore the feasibility of applying key features of the multi-unit auction to proxy buyer/seller marginal valuations of the *attributes* of a job.

## Multi-Attribute Labor Market Auction Model

A first-price sealed bid auction was designed to provide an environment where buyers and sellers could submit offers and/or counteroffers on the attributes of a hypothetical job. The experimental auction is designed to include constraints that are inherent within a military labor market setting. These constraints include (1) forced market convergence, that is a hypothetical job must be filled and (2) control of a seller's value, where seller value can be defined as a measure of the marginal productivity (marginal cost) of a seller. The labor market auction model is flexible in that experimental parameters can be set by the experimenter to examine the effect of bidding behaviors under varying constraints and/or rules.

In order to prevent individuals from bringing in egalitarian, altruistic, or other preferences from everyday social life into the experiment, jobs are assigned generic names and each level of compensation for a given job is referred to as an 'attribute'.

The user interface multi-attribute auction model user interface (see Figure 1 below) allows the auction participants to view bidding history and earnings information, as well as participate in one or more auctions in a given time period.<sup>1</sup>

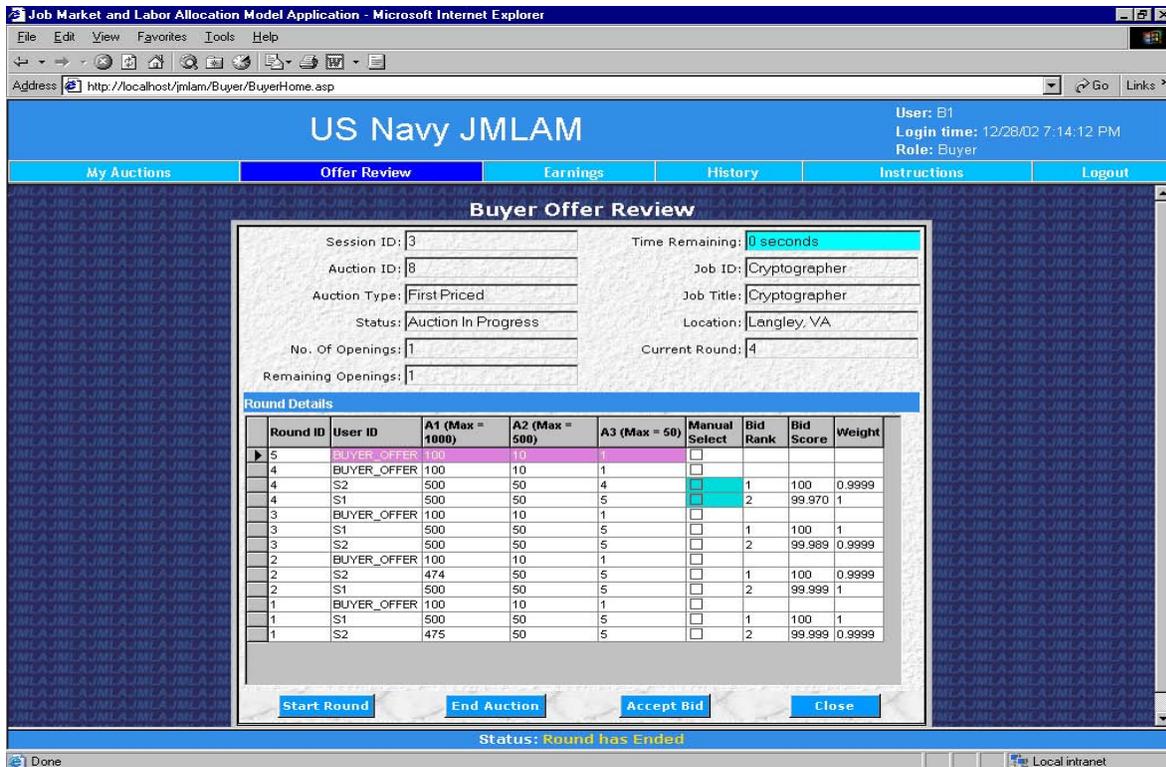


Figure 1. Labor market auction interface.

The auction is designed so that each seller can sell a single unit of labor, while the buyer can purchase one or more units of labor. The maximum number of labor units that the buyer is willing and able to purchase is unknown to the seller. Subjects are informed that at least one job opening is available to be filled, but, that the buyer is not obligated to accept a bid from any seller. The buyer's objective is to maximize the difference between the buyer's reserve and the seller offers. Earnings only accrue to the buyer if he or she accepts a seller's offer and the offer is less than the buyer's reserve. Buyer's earnings, therefore, increase the greater the difference between the buyer's reserve, the accepted seller's offer, and the number of seller's bids the buyer accepts.

Sellers submit bids on three hypothetical components of a compensation package or attributes of a job. Associated with each attribute is a reserve price and depending on the experimental parameters (see Tables 1 and 2), the reserve prices vary across sellers and auctions. Sellers know their reserve prices however, sellers do not know other sellers reserve prices or the distribution of reserve prices. Seller's earnings are determined by the difference between the composite attribute reserve price and the

<sup>1</sup> Experiments discussed herein, however, have been limited to the participation of subjects in only one auction at a time.

submitted bid for each attribute. The greater the difference between the composite reserve price and the submitted bid, the greater the seller's earnings. Earnings only accrue to the seller if the seller wins an auction. The probability of winning an auction diminishes the greater the difference between the seller's reserve price and his or her submitted bid. Bidding strategy is further complicated by the fact that each seller is assigned a "seller value"; a continuous variable constrained to take on the value between 0 and 1 and is used as an indicator of the seller's marginal product. The greater the seller value, the greater the assumed productivity of the seller. Each seller knows his or her seller value, but other seller values and/or the distribution of seller values are unknown to individual sellers. The buyer, however, knows all seller values and the distribution of those values.

The probability of winning an auction, therefore, is a function of the seller bid and seller value. Given two sellers;  $s_1$  and  $s_2$ , with seller values of 1 and .5 respectively and who submit identical bids, the seller with the higher seller value will win the auction, in this case  $s_1$ . A scoring rule, discussed below, that considers the seller value relative to the submitted bids is used to determine the most efficient or optimal bid.

### Multi-Attribute Labor Market Auction Model: Scoring Rule

Multi-unit auctions have been used to auction off Federal Communications Commission (FCC) licenses, transportation services, and delivery routes (Matsui & Watanabe, 2003). A key feature of multi-unit auctions is that they allow the buyer to submit a single bid on a combination of units of the buyer's choosing. Numerous auction algorithms have been used to determine winners in multi-unit auctions.

Typically, optimization algorithms used in multi-unit auctions are designed to maximize the auctioneer's revenue, maximize allocative efficiency, and/or determine location specific prices (McCabe, Rassenti, & Smith, 1991). Examples of the use of such algorithms can be found in the FCC spectrum, airline slots, gas pipeline networks, and sales routes auctions (Ledyard, Olson, Porter, Swanson, & Torma, 2002; McCabe et al., 1991). The primary difficulty with multi-unit auctions is evaluating the winner of a given auction. The ability of the buyer in the labor auction environment to evaluate each seller's bid is likely to be computationally intractable.

The optimization algorithm used on the multi-attribute labor market, rank orders sellers as a function of cost (bid) relative to the seller value (marginal productivity). Data Envelopment Analysis—Cross Efficiency (DEA-CE) is the scoring rule used to determine the auction winner(s).<sup>2</sup> The primary benefit of DEA-CE as a scoring rule is that DEA-CE can be used as a measure of the efficiency of a decision making unit (DMU) and that the DEA-CE scoring rule minimizes the number of ties across feasible bids. DEA-CE assigns a rank based on the weighted average of inputs to outputs, where the weights are chosen so as to make  $DMU_i$  (the  $i$ th decision making unit, or equivalently the  $i$ th seller) look as good as possible relative to other DMUs. The DEA-CE scoring algorithm uses the bids on the attributes,  $A_1$ ,  $A_2$ , and  $A_3$  as the inputs and the seller value as an output. The attributes can take on the value of any whole number

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<sup>2</sup> DEA-CE Game Theoretical Approach is discussed in J.J. Rousseau and J.H. Semple, Two-Person Ratio Efficiency Games, *Management Science*, Volume 41, Issue 3 (March 1995), 435–441.

between 1 and infinity, whereas the seller value is a continuous variable constrained to lie between 0 and 1. If the seller value is interpreted to be a productivity measure, then the higher the seller weight the greater the productivity of the seller. Similarly, if the seller value is defined as a cost efficiency measure, the closer the seller value is to 1 the greater the cost savings realized by the buyer.

Defining outputs and inputs as seller value,  $v$ , and attribute bids,  $a$ , respectively, then an efficiency measure of outputs to inputs can be defined as follows:

$$\sum \mu_i^* v_i / \sum \theta_i^* a_i$$

where  $\mu$  = the weight associated with seller value.

$\theta$  = the weight associated with the attribute bids.

$0 < \mu, \theta \leq 1$

The result of DEA is the weighted average of outputs to the inputs, where the weights are chosen for each DMU such that the ratio of outputs to inputs of DMU<sub>i</sub> is as efficient as possible relative to any other DMU.<sup>3</sup> A DMU is classified as 100 percent efficient if it is not possible to improve the weights of any of its input or outputs without adversely affecting any other DMU. Once the best weights for a given DMU have been obtained, the cross efficiency of a DMU is determined by comparing DMU<sub>i</sub> with DMU<sub>j</sub>, using DMU<sub>j</sub>'s weights.

DEA only allows us to compare efficiencies across DMUs. The cross efficiency matrix compares each DMU<sub>i</sub> (bidder) with all other DMU<sub>j</sub>s, using the weights of DMU<sub>j</sub>s. In other words, DEA-CE applies the weights of each DMU<sub>j</sub> to the input-output data of DMU<sub>i</sub>. In order to determine a ranking measure, a cross efficiency matrix is used to calculate the average cross efficiencies.

An example of a cross efficiency matrix is shown in Table 1, below:

**Table 1**  
**Rated seller**

Rating Seller	Average CE Score			
	1	2	3	
1	E11	E12	E13	A1
2	E21	E22	E23	A2
3	E31	E32	E33	A3
Average CE score	CE1	CE2	CE3	
Rank	1	2	3	

<sup>3</sup> The weights of the inputs are determined simultaneously.

The leading diagonal gives the DEA efficiencies. The cells above and below the leading diagonal are  $DMU_i$ 's cross efficiency using  $DMU_j$ 's weight. For example E13 is the cross efficiency of seller 3 using seller 1's weights. Average cross efficiencies can be obtained by summing across rows or columns; CE3 is the average cross efficiency for seller 3. The average CEs can then be converted to a rank. The closer the average CE score is to 100 the higher the rank accorded a given DMU (seller). It is important to note that DEA scores will always be higher than average cross efficiency scores, as DEA chooses weights that make a given DMU as efficient as possible.

The CE matrix is obtained from running the DEA model embedded in multi-attribute labor market model. The scoring rule provides to the buyer the seller ranks and seller rank scores. The auction rules dictate that given two sellers, seller 1 with a rank score of 1 and seller 2 with a rank score of .95, the buyer must choose seller 1 as the auction winner. A general interpretation of the rank score is that the ratio of the productivity (cost) of the  $i$ th seller to the  $i$ th seller's bid is ranked against the productivity (cost)/bid ratio for all the other bidders. For example, given two sellers,  $s_1$  and  $s_2$ , assuming that  $s_1$  and  $s_2$  submit identical bids, but  $s_1$  has a relatively higher seller value, then  $MPL_1/bid_1 > MPL_2/bid_2$ , and  $s_1$  will receive a higher rank score, (likewise  $MC_1/bid_1 > MC_2/bid_2$ ).<sup>4</sup>

Assuming that the buyer is willing and able to accept a bid, the buyer chooses the seller with the highest rank. If the buyer can purchase multiple labor units the buyer simply chooses the  $n$  highest ranked sellers until the number of available units is exhausted.

## Experimental Design

Using the multi-attribute labor market auction software, parameters for two sets of experiments were chosen. The experimental parameters were calibrated using a volunteer subject pool. Tables 2 and 3 describe the parameter settings used in Experiment 1 and Experiment 2, respectively.

The experimental parameters adopted in Experiment 1 were largely designed to provide a benchmark against which to compare results from Experiment 2. In Experiment 1, seller values remained fixed across sessions and auctions. In approximately one-half of the auctions subject reserve prices were fixed across auctions. In the cases that reserve prices varied, subjects were randomly assigned to one of the following three groupings; (10, 5, 2), (13, 7, 3) or (18, 9, 4).

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<sup>4</sup> MPL - Marginal Product of Labor. MC – Marginal Cost

**Table 2**  
**Experiment 1 parameters**

Seller Value	1
Reserve Prices, N = 360	Attribute 1 (A1) = 10, Attribute 2 (A2) = 5, Attribute 3 (A3) = 2 or (10, 5, 2)
Reserve Prices (A1, A2, A3), N = 408	(10, 5, 2) or (13, 7, 3) or (18, 9, 4)
Job Openings	2, unknown to the seller
Seconds for Play	90-105 seconds
Conversion Rate	.05, \$1 Experimental Dollar = \$0.05 USD
Maximum Number of Rounds	10, unknown to seller
Buyer Reserve (A1, A2, A3)	(100, 50, 25), unknown to a seller
Maximum Number of Sellers	9, average subjects per session was 7

Approximately 60 subjects participated in the first experimental session. Each subject group participated in 2 sessions of 10 auctions each, with a total sample size of 768 observations. In general, the two highest ranked sellers were chosen as auction winners. While the auction is designed to enable multiple rounds, winners were always assigned in the first round. In general, sellers submitted bids very close to their reserve; typically within \$1–\$3 experimental above their reserve and in some cases sellers bid their reserve on at least one attribute. Submitting bids equal to seller reserves on at least one attribute may be attributed to the fact that sellers were unaware of the buyer's reserve prices. Sellers may have adopted a strategy of bidding at the reserve on a least one or more attributes in an effort to increase the probability of winning a given auction

In Experiment 2 subjects were asked to participate in 3 sessions, for a total of 810 observations. Experiment 2 subjects were assigned differing seller values and corresponding reservation prices in a random order. Higher seller values imply higher productivity and, therefore, higher associated reserve prices were assigned to sellers with higher seller values. Based on the bidding behaviors observed in Experiment 1, the seller value/reserve prices were set so that there would be an equal probability of winning an auction, independent of the seller value/reserve price combination.

**Table 3**  
**Experiment 2 parameters**

Seller Value	.6, .8 or 1, randomly assigned session/auction/subject
Reserve Prices (A1, A2, A3)	(13, 10, 6) if seller value = .6
Reserve Prices (A1, A2, A3)	(16, 12, 8) if seller value = .8
Reserve Prices (A1, A2, A3)	(20, 15, 10) if seller value = 1
Job Openings	2, unknown to the seller
Seconds for Play	60 seconds
Conversion Rate	.10, \$1 Experimental Dollar = \$0.10 USD
Maximum Number of Rounds	10, unknown to seller
Buyer Reserve (A1, A2, A3)	(100, 50, 25), unknown to the seller
Maximum Number of Sellers	9, average subjects per session was 7

## Auction Results

At this stage of the research it is uncertain as to whether subjects focus on the summed value of their bids or a composite bid, or consider each attribute bid independently. To this extent, analysis on composite bidding behavior and individual attributes are provided in the Figures below. Median and minimum bids are examined as a cursory insight into bidding behaviors in Experiment 1 and 2. The median bid, relative to a mean bid, is examined to mitigate the effect of outliers.

The base case, Experiment 1, all seller values were fixed at “1.” In approximately one-half of the auctions, the reserve prices were fixed across subjects, with subjects randomly assigned variable reserve prices in the remaining auctions. Figure 2 reflects the median difference in the composite reserve prices and composite bids. In the initial auctions the median difference in the subject groups is opposite of what we had expected and surprisingly large. One would expect that the median bid in the “same group” would be relatively lower than the median bid in the “different reserve group,” particularly, since the reserve prices for two-thirds of the subjects was significantly larger than the “same reserve group.” Over successive auctions, however, the median difference between the two groups begins to converge.

Figure 3 shows the minimum difference in the composite bids and reserve prices. For the “same reserve price” group at least one subject submitted a composite bid that equaled his or her reserve price. Bids submitted at reserve may imply that subjects placed a greater value on simply winning the auction relative to any earnings potential; alternatively, subjects may be searching for information on possible winning bid combinations. In contrast, the minimum bid in the “different reserve group” was consistently above the minimum composite reserve of the “same reserve group” (see Figure 3). This effect could be attributed to biases in the experimental design. In Experiment 1, there was no randomization in the order subjects played the auctions. In all cases, subjects participated in the “same reserve price” auctions and then the “different reserve price” auctions. This non-randomization of experiments may have

provided information to the subjects regarding the distribution of reserve prices across subjects, thereby driving the minimum bids upward.

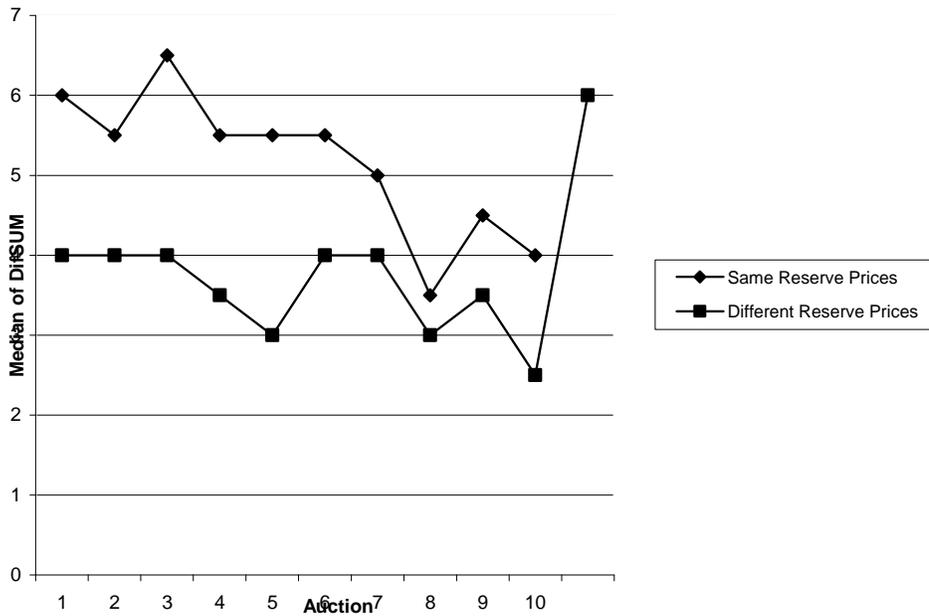


Figure 2. Experiment 1 median of difference in composite bids.

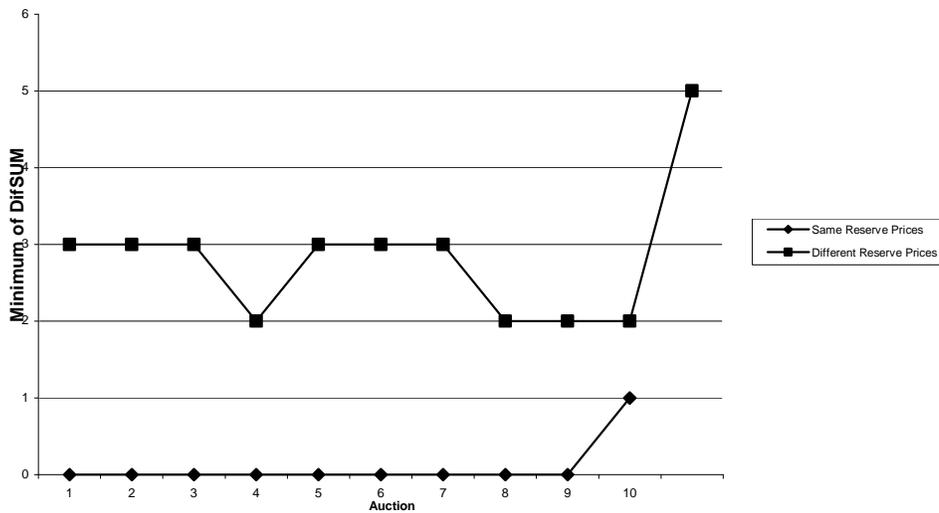


Figure 3. Experiment 1 minimum of differences in the composite bid.

In Experiment 2, four groups of subjects participated in three auction sessions.<sup>5</sup> Each auction session consisted of 9 auctions for a total of 108 auctions completed. In each auction, two units of the hypothetical job were available; therefore, for each

<sup>5</sup> Group 1 participated in auctions 1–27, Group 2 participated in auctions 28–53, Group 3 participated in auctions 54–80, and Group 4 participated in auctions 81–108. The quadrants, denoted by vertical lines, in the Experiment 2 graphs are mapped to group specific bidding behaviors.

auction the two highest ranked bidders were awarded the units. In contrast to Experiment 1, subjects in Experiment 2 were assigned varying seller values; with each seller value assigned a corresponding set of reserve prices.

In Figures 4 and 5 we look at the differences in the median and minimum composite bids. A pattern of convergence to the composite reserve prices is observed across all four groups. In general, the minimum composite bid allowed subjects to potentially earn \$1 Experimental. Interestingly, at least one subject per auction submitted his or her reserve price, possibly meaning that some subjects may have been searching for information on winning bid combinations.

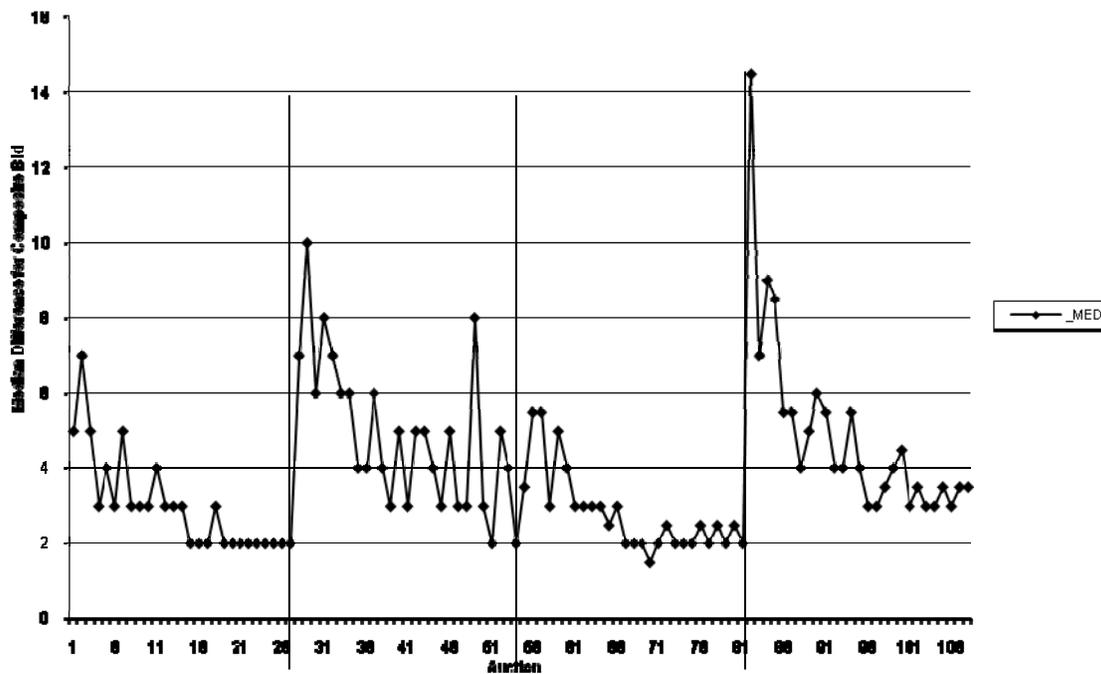
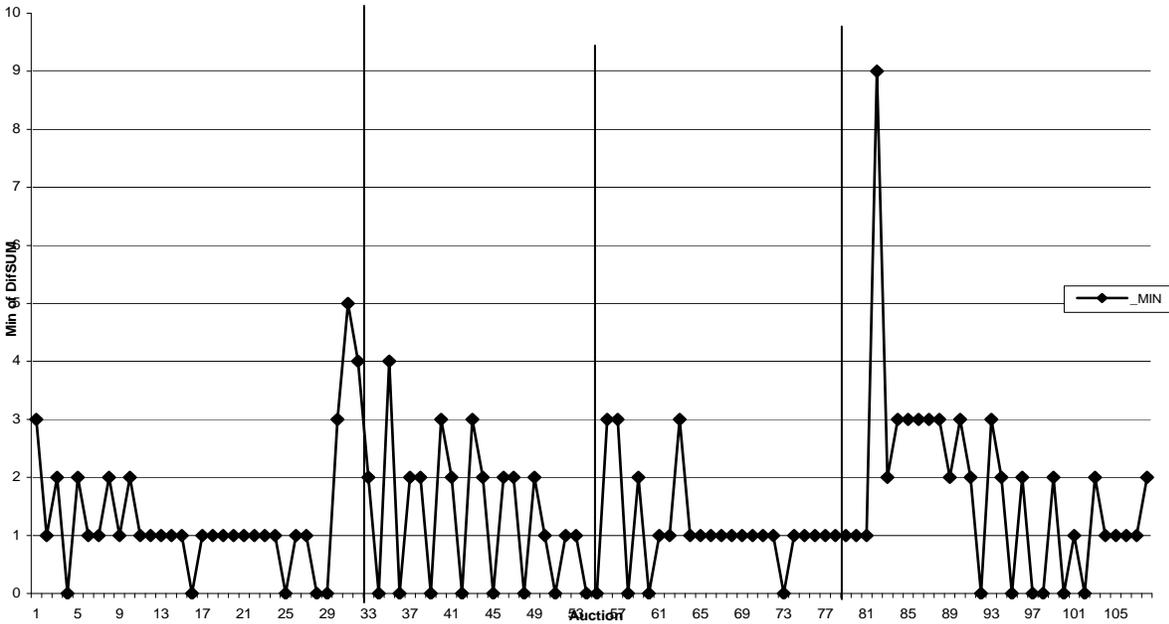


Figure 4. Experiment 2 median difference of composite bid.



**Figure 5. Experiment 2 minimum of composite bid difference.**

In Figures 6–11 we decompose the bidding behavior of the four groups by attributes. There appears to be a distinct demarcation between Groups 1 and 3 and Groups 2 and 4. Groups 1 and 3 have a more frequent relative tendency to bid their reserve on at least one attribute. As compared to the median bids for Groups 2 and 4, where, in general, the median was above the attribute reserve price across all attributes.

A comparison of the median bid between A1 and A2 indicates that in the initial auctions subjects were more likely to bid relatively lower on A2 than A1; however, the median bid for A3 remained on average \$1 Experimental above the subjects reserve. The subject focus on A1 can perhaps be attributed to relative weight of A1’s reserve price. It may be the case that subjects perceive that attributes with higher reserve are given more weight in determining the auction winner. If subjects believe that A1 is the influencing attribute, then a winning bidding strategy of bidding relatively lower on A1 and higher on A2 and A3 may exist. In Experiments 1 and 2 subjects were not (de)briefed on possible winning bidding strategies nor did the experiments allow for experience bidders.

Submission of a bid equal to the reserve, the minimum, occurred more frequently in Groups 1 and 3. Subjects in Group 2 appeared to be less likely to submit a minimum bid on A1 and A2, but more likely to submit a minimum bid on A3. Of the three attributes, Group 2 was more likely to bid the minimum on A1 and A3, but in only one auction, auction 50, do we observe a minimum bid on A2. Overall Group 2 had higher earnings relative to the other groups. We postulate that the higher relative earnings of Group 2 and hence their bidding behavior may have resulted from a break in protocol during the training sessions.

The training sessions are designed to familiarize the subjects with the auction software and rules. For purposes of training, the buyer accepts the two highest ranked feasible bids, regardless of the bid value. In the Group 2 training session, the group as a whole submitted very high bids, with the experimenters announcing the earnings. The announcement of a high earnings may have induced the group to bid relatively more aggressively.

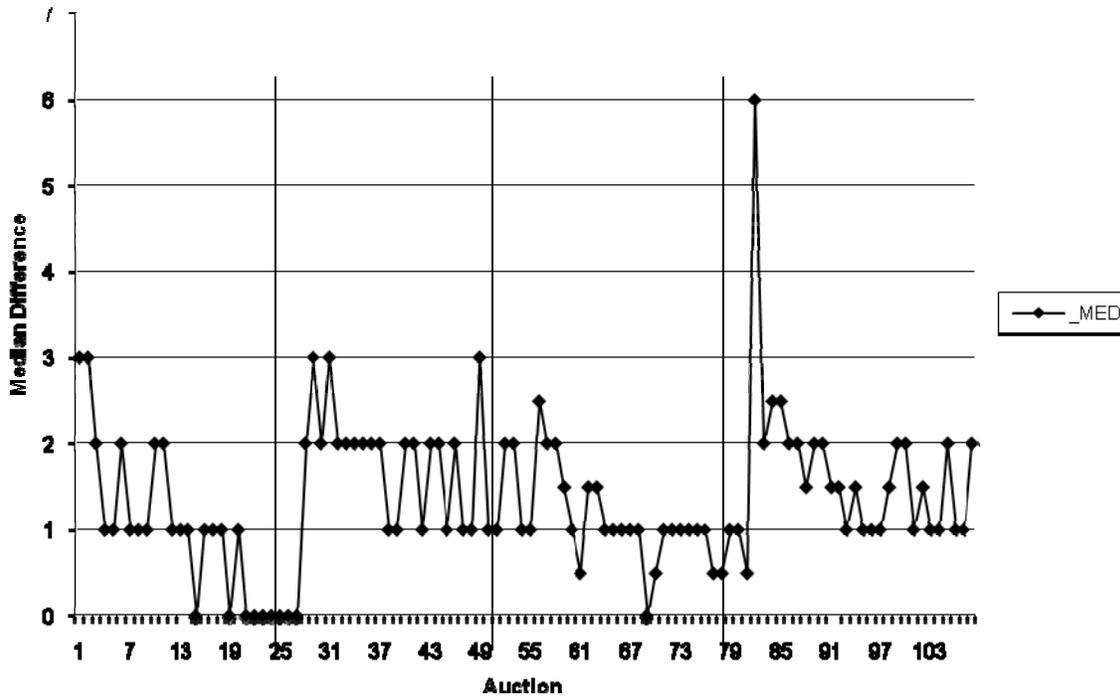


Figure 6. Experiment 2 Median difference between reserve price and bid for A1.

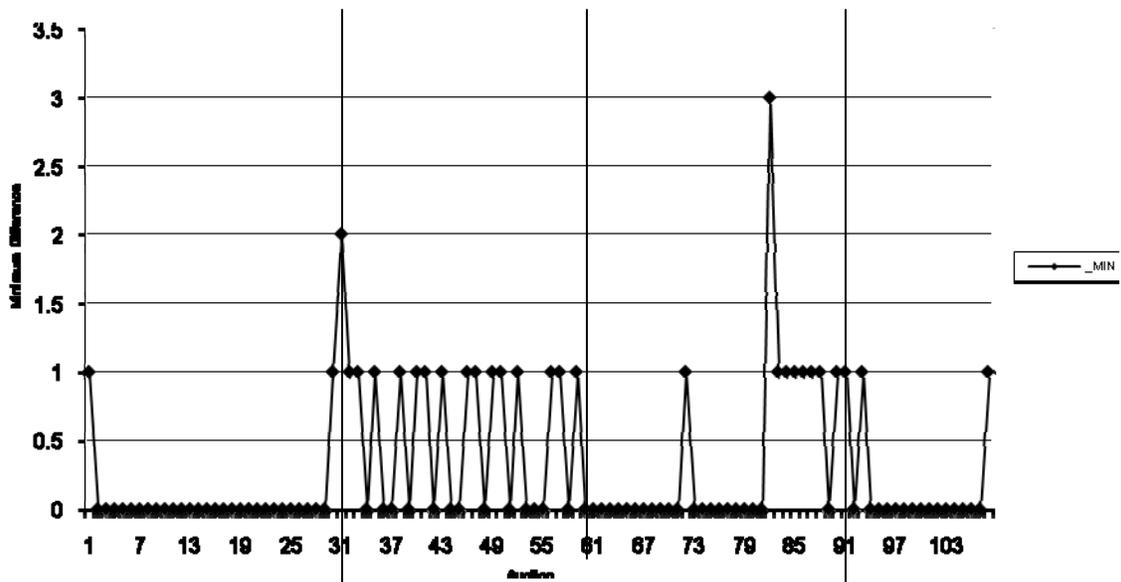


Figure 7. Experiment 2 minimum difference between reserve price and bid for A1.

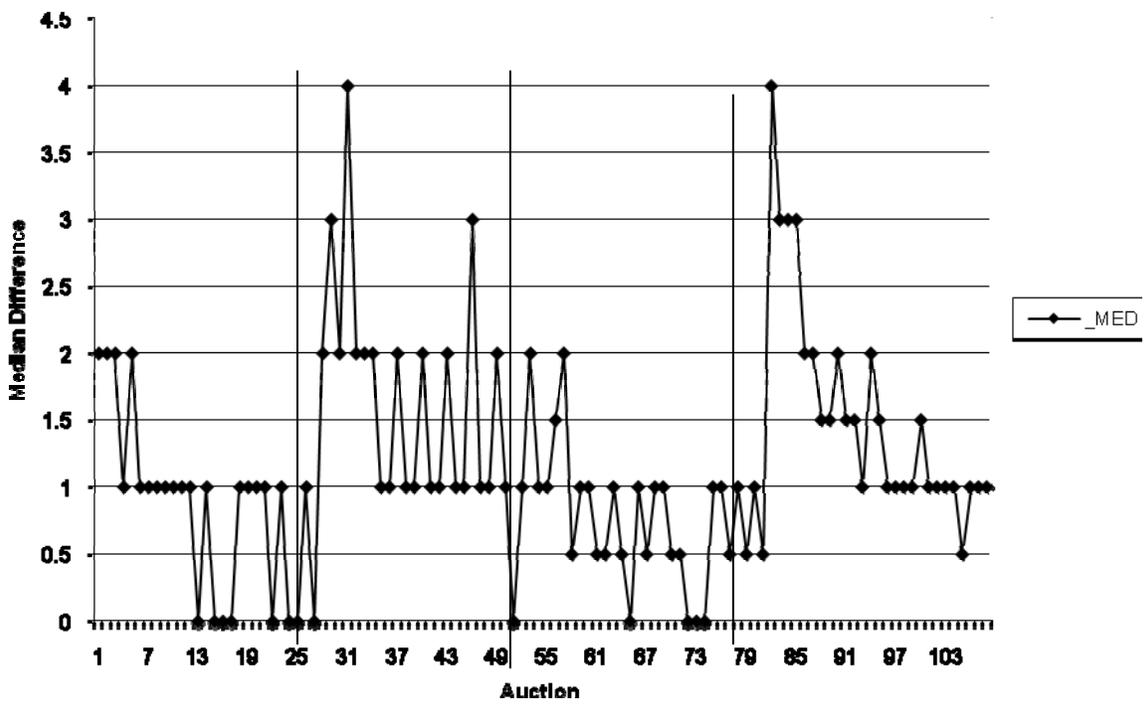


Figure 8. Experiment 2 median difference between reserve price and bid for A2.

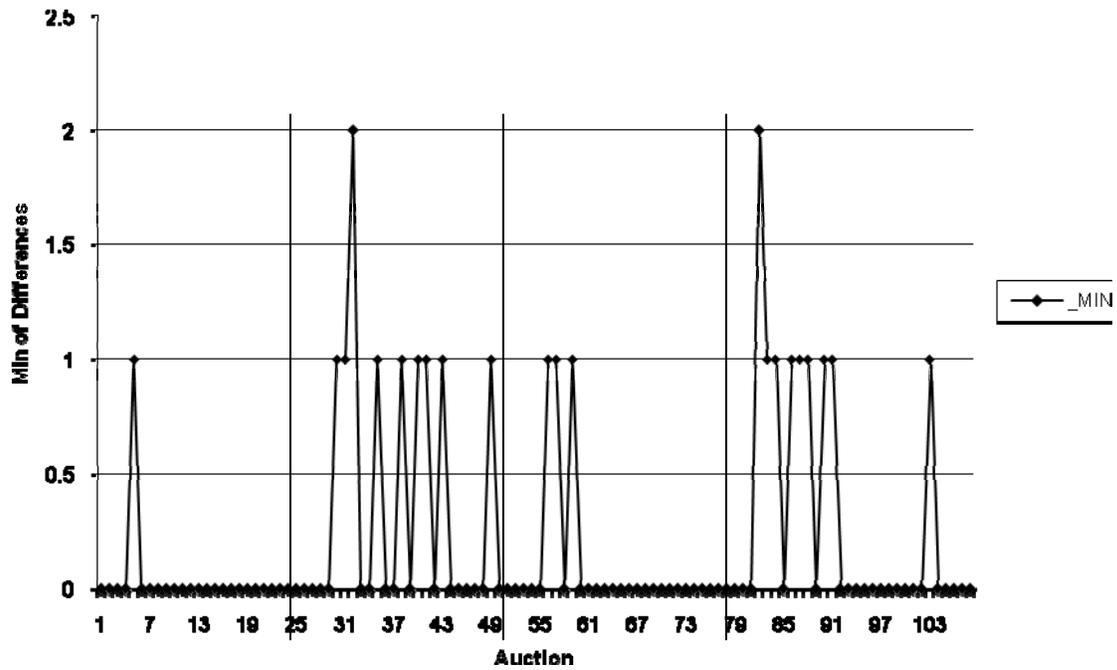


Figure 9. Experiment 2 minimum value of differences between reserve price and bid for A2.

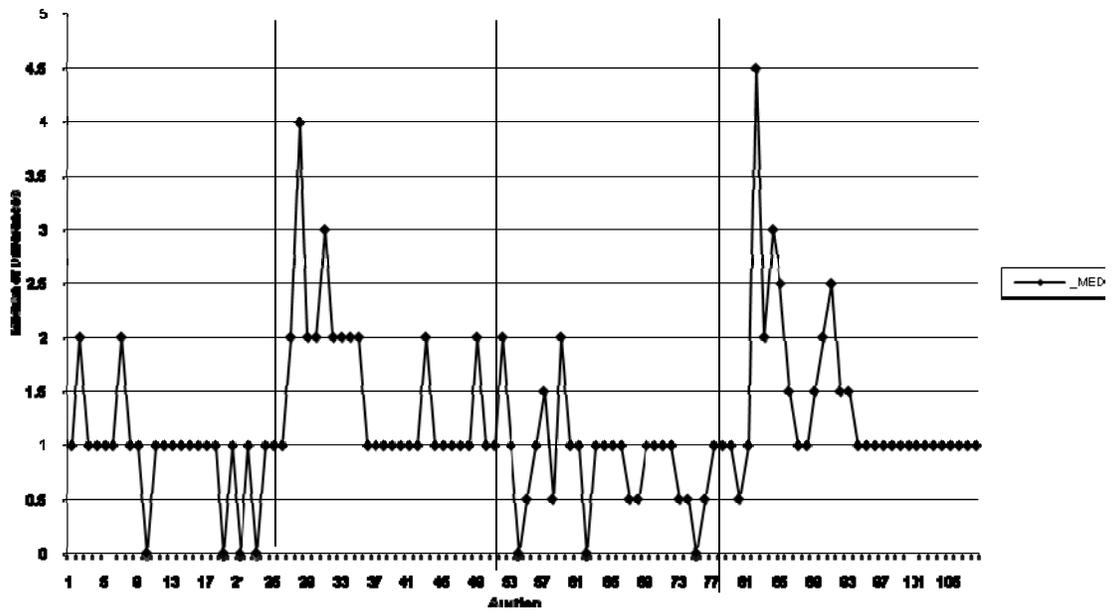


Figure 10. Experiment 2 median of differences between reserve price and bid for A3.

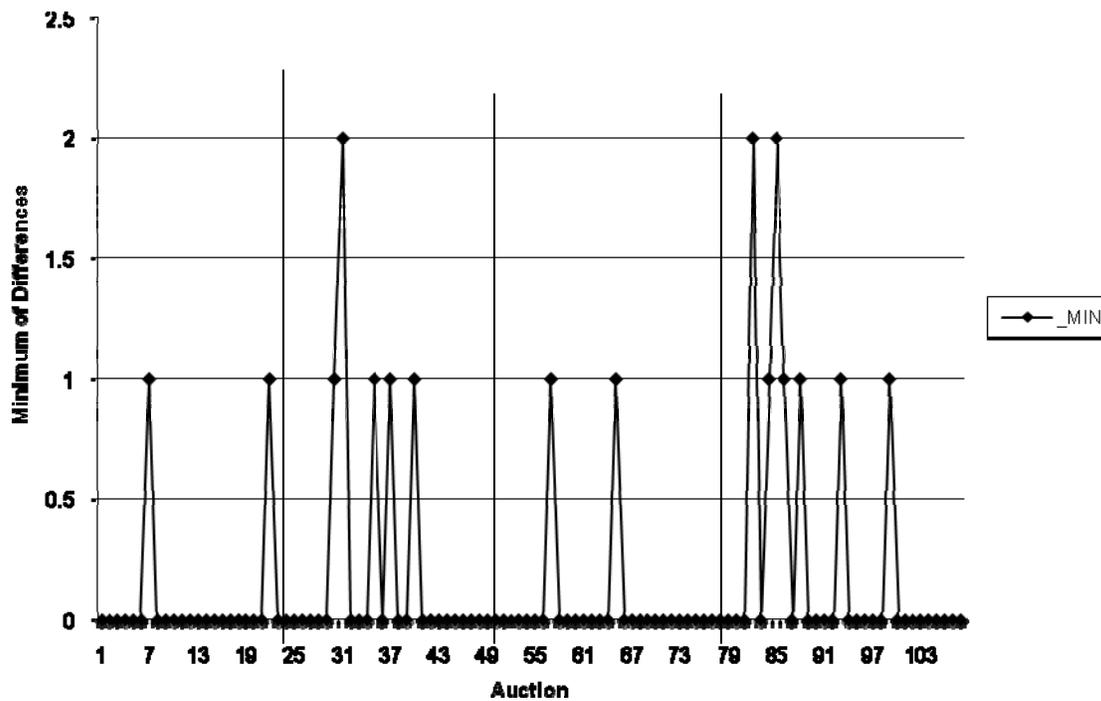


Figure 11. Experiment 2 minimum of differences between reserve price and bid for A3.

The combination of seller value and associated reserve price was established through successive testing of the auction parameters. The seller value-reserve price combination was set such that each seller in the auction had an equal opportunity of winning. In order to give sellers with low values an equal opportunity to win auctions the high value sellers were restricted to higher reserve values.

The difference in composite bids to reserves by seller value for the composite bid is illustrated in Figure 12. As expected, the greater the seller value the greater the difference in the composite bid, as the high value bidders attempt to exploit their high productivity. However, the relatively larger difference in the composite bid is observed only in the early auctions. Regardless of the seller value, over successive auctions the difference in composite bids begins to converge towards the reserve. While participants had no a priori knowledge of the distribution of seller values or reserve prices, it is likely that over successive plays subjects inferred the distribution of these parameters, thereby influencing a minimum bidding strategy.

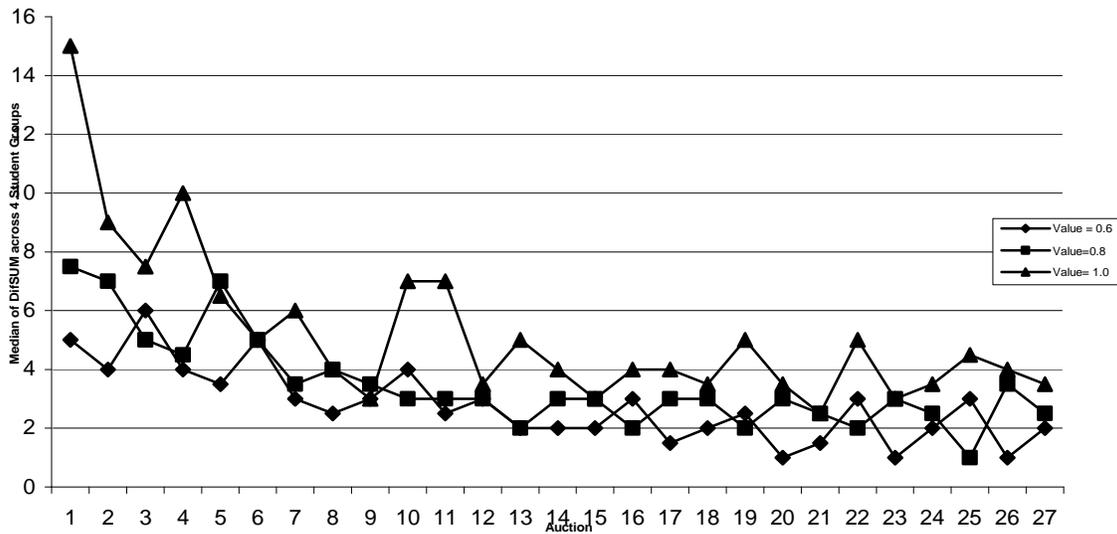


Figure 12. Median of difference of composite bid by seller value.

## Conclusion

Two experiments were executed to observe bidding behavior in a multi-attribute auction setting over varying reserve prices and seller values. On average 7 subjects participated in each experimental session or 27 auctions. Interestingly, convergence of subject bids to individual reserve prices generally occurs within five auctions and even with as few subjects/bidders as seven.

While theoretically it can be shown that a first-price open out cry auction quickly converges to subject reserve prices, the first-price sealed bid multi-attribute auction design addressed in this paper also quickly converges to the subject reserve. Interestingly, the rapid convergence occurs even in cases were seller values differ across auctions.

The convergence of bids to the subject reserve price indicates that the application of a multi-attribute auction results in bids such that relative efficiencies as measured by the productivity (cost)/ bid ratio is achieved. Applications of a multi-attribute auction to labor markets, where participants bid on multiple components of a compensation package show promise in ascertaining buyer/seller marginal valuations of a job.

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